Errata to The Analysis of Algorithms, Second Printing. 8–27–2006

Usually just the corrected segment of text is given. Negative line numbers indicate the number of lines from the bottom.

p 2, l 3. (Only for \(n \geq 5 \times 10^{17}\) does \(n^{0.1}\) become noticeably larger than \(\log n\).)

p 18, ex. 4.

\[
2 \ln\left(1 + \frac{t(-\ln(1+p))}{\ln(1-p)}\right) = \left(\frac{\ln 2 + t(-\ln(1-p))}{\ln 2}\right)^{\ln 2/(\ln(1-p))}.
\]

p 20, ex. 2. Show that the number of times you go around the loop in the Euclidean Algorithm (Algorithm 1.5) is no more than \(2 \log n\).

p 21, l 12 (\(m\) in our example)

p 27, l 2. A \textit{triangular matrix} is one in which all the elements above

p 27, ex. 2. previous exercise can be written as

p 36, l 1, Step 5 is done the same number of or one less time than Step 4 each time

Step 2 is done.

p 49, l 7. order. The first element in each file has subscript

p 49, l 9. once again the first remaining element has subscript

p 49, Step 3. Steps 3–5 and 6–8 are

p 52, eq. (105). \(= 1 + \max\{C_{m,n/2-2^{r-1}} + m, t + C_{m-1,n/2} + m - 1\}\).

p 53, eq. (106-107) \(C_{m,n} = 1 + \max\{C_{m,n/2-2^{r-1}} + m, t + C_{m-1,n/2} + m - 1\}\) provided \(n\) is even, \(m \geq 2\), and \(n \geq 2m + 2\). Now comes a clever portion of the proof, a crucial observation: by eq. (103) \(1 + \max\{C_{m,n/2-2^{r-1}}, t + C_{m-1,n/2} - 1\} = C_{m,n/2}\), so we have

p 53, l 9. left side of eq.

p 55, ex. 3. \(C_{m,2m+1}\)

p 55, l 20. A predicate \(P\) is called \(\rightarrow\)-complete if, whenever \(P(x)\) is \textit{true} for all \(x\) in \(\delta^+(y)\), then \(P(y)\) is also \textit{true}. (In particular, \(P(y)\) is \textit{true} when \(\delta^+(y)\) is empty.

p 55, l 10. A relation is \textit{confluent} if \(x \uparrow y\) implies \(x \downarrow y\). Confluence is important because, in a Noetherian confluent relation,

p 61 l 1. For example, the formula \(\sum_{0 \leq i \leq n} a_i\) is not in closed form, because the summation sign stands for the addition of \(n + 1\) elements.

p 65, l 10. The second sum

p 67, No period at the end of equation 31.

p 72 l 19. in Steps 3 and 6 of Split.

p 74, TABLE 2.1
<table>
<thead>
<tr>
<th>Position</th>
<th>Input</th>
<th>Intermediate</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>A A A A A</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>G</td>
<td>B A A A A</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>C B B B B</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>H</td>
<td>A C C C C</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>F F F F F</td>
<td>F</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>G G G G G</td>
<td>G</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>I I I I I</td>
<td>H</td>
</tr>
<tr>
<td>7</td>
<td>J</td>
<td>J J J J J</td>
<td>J</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>H H J J J</td>
<td>J</td>
</tr>
<tr>
<td>9</td>
<td>J</td>
<td>J J J J J</td>
<td>J</td>
</tr>
</tbody>
</table>

This time with \(l = 6, r = 8\). Split selects the element in position 6 (I) as the splitting element and produces the results shown in the fourth intermediate column of Table 2.1. Quicksort (level 2) calls Quicksort (level 3) with \(l = 6, r = 6\) (Quicksort (level 3) returns from Step 1), and then with \(l = 8, r = 8\) (Quicksort (level 3) returns from Step 1). Then (level 2) and (level 1) finish to give the results in the output column of Table 2.1.

What is the worst-case time for the variation of Quicksort?

If the \(q_i\) are easier to compute than the \(p_i\) (as they were in Section 17), for example, the degree of \(R(i)\) is less than that of \(D(i)\). For example, the method in this section

\[
\int_0^\infty e^{-t}t^{x-1} dt - \Gamma_m(x) = \int_0^\infty e^{-t}t^{x-1} dt + \int_0^m \left(e^{-t} - \left(1 - \frac{t}{m}\right)^m\right) t^{x-1} dt
\]

Suppose, for example, that you need to simplify \(\sum_i \binom{n+i}{i} x^i\), where \(|x| < 1\). The smudges on this page appear to be on the plate. If so, a new plate should be shot.

\[\omega^{n/2}(\omega^{-1/2} + \omega^{1/2})^n = \omega^{n/2}(2\cos(\pi/3))^n.\] Likewise,
\[
\sum_i \binom{m}{i} \binom{m-i}{i+j-m} 2^{-2i-j}.
\]

p 130, ex. 2.

p 133, l 2 and eq. 180. In general, the Stirling number \( \left[ \begin{array}{c} n \\ k \end{array} \right] \) is the sum of all the products of \((n - k)\) different integers taken from 1 to \(n - 1\); that is,

\[
\left[ \begin{array}{c} n \\ k \end{array} \right] = \sum_{1 \leq i_1 < i_2 < \cdots < i_{n-k} \leq n-1} i_1 i_2 \cdots i_{n-k}.
\]

p 134, l 3 and eq. 186. In general, the Stirling number \( \left\{ \begin{array}{c} n \\ k \end{array} \right\} \) is the sum of all the products of \(n - k\) integers taken from 1 to \(k\); that is,

\[
\left\{ \begin{array}{c} n \\ k \end{array} \right\} = \sum_{1 \leq i_1 \leq i_2 \leq \cdots \leq i_{n-k} \leq k} i_1 i_2 \cdots i_k.
\]

p 135, l 12. One of these ways, however, has one part empty.

p 144, eq. 223.

\[
\sum_i \binom{r}{i} \binom{s + i}{n} (-1)^i = \sum_{i \geq 1} \frac{s!(s+1)!(-r)^i}{i!(s-n)!(s-n+i)^i}.
\]

p 145, l 5. Many results on hypergeometric functions can be generalized to basic hypergeometric functions, which are obtained

p 146, eq. 243.

\[
N() = N + \sum_{1 \leq i \leq k} (-1)^i \sum_{1 \leq j_1 < j_2 < \cdots < j_i \leq k} N(A_{j_1} A_{j_2} \cdots A_{j_i}).
\]

p 146, eq. 244.

\[
\frac{N(A_1 A_2 \cdots A_i)}{N} = \frac{N(A_1) N(A_2) \cdots N(A_i)}{N^i},
\]

p 147 eq. 245.

\[
\phi(n) = n + \sum_{1 \leq i \leq k} (-1)^i \sum_{1 \leq j_1 < j_2 < \cdots < j_i \leq k} \frac{n}{p_{j_1} p_{j_2} \cdots p_{j_i}}.
\]

p 147 eq. 246.

\[
\phi(n) = n \left( 1 - \frac{1}{p_1} \right) \left( 1 - \frac{1}{p_2} \right) \cdots \left( 1 - \frac{1}{p_k} \right).
\]

p 147, l 6. pigeonhole

p 150, l 10.

\[
\min_{x_0 \leq x \leq x_1} \{ g_L(x) \} \leq y \leq \max_{x_0 \leq x \leq x_1} \{ g_U(x) \}.
\]

p 150, l 2. where \(x\) will be required to be in some range \(x_0 \leq x \leq x_1\) and \(U\) (alternately \(L\)) is an upper (lower) bound on \(f^{(n+1)}(x)/(n+1)!\) in the range of \(x\).

p 151, eq. 15.

\[
e^x \geq 1 + x \quad \text{for} \quad x \geq 0 \quad \text{and} \quad e^x \leq \frac{1}{1-x} \quad \text{for} \quad 0 \leq x < 1.
\]

15
4

p 151, l 11. for some c in the range $0 \leq c \leq x$. Using Principle 9 (with $n = 0$) gives
$p 151$, eq. 20

$$0.9039 \leq (1 - 1/100)^{10} \leq 0.9049.$$  

p 153, l 2. $n\sqrt{N}$, testing $1 + (n - 1)\sqrt{N}$, . . . , $n\sqrt{N} - 1$. (Actually, there is no need
to test against $N$ because we know the value is less than or equal to $N$).

p 154, l 7. The trees for $(^d_0)$ and $(^d_1)$ each contain a single node.

p 154, l 15. For testing values up to $N$ we need a tree with at least $N$ leaves.

p 154, l 12. a tree has at least $N$ leaves.

p 154, l 6. $h$ items by doing at most $d(\binom{h}{N}) = h + \binom{h}{h} = h + 1$ tests.

p 159, l 19. $a^m_{m+1} > 0$ we have

$$a^m_{m+1} = a^m_{m+1} + a^m_{m+2} + a^m_{m+3} + \cdots = (f(r) - \sum_{0 < i < m+1} a^m_{i+1} a^m_{i}) / r^{m+2},$$

which is finite. This establishes eq. (45) with the $C$ for eq. (23) equal to $a^m_{m+1}/2$

p 163, eq. 71.

$$\left(1 - \frac{1}{100}\right)^{10} = e^{-1/10} \left[1 + O\left(\frac{1}{100}\right)\right]$$

p 164, ex. 7. Show that ($n$ is the variable)

p 165, l 4. at $x_0$ if $\lim_{n \to \infty} [f(x_0) - s_n(x_0)] = 0$.

p 169, l 15. Backtracking is a method of organizing a search through

p 173, l 22. The probability that a clause does not contain the literal

p 175 l 18. for another example).

p 175, l 10. Let’s use $r_k$ for the probability that $P^j_{k-1}(false, . . . , false) = true$.

p 175, l 9. only literals for the first $k$ variables and all the literals are positive, where

$p 176, l 2.$ that a clause does not have this form.

p 177, l 9. Now eq. (102) is a decreasing function of $i$ [because $-(1 - p)^{-1}$

p 179, eq. 118.

$$2^{-\frac{\ln(1 + (\frac{\ln(1 + 1/p)}{-\ln(1 + 1/p)})^\ln 2 / (-\ln(1 - p))}{\ln 2 + t(-\ln(1 - p))}} = \frac{\ln 2}{\ln 2 + t(-\ln(1 - p))}$$

p 179, eq. 119.

$$N_i = 2^2 \left(\frac{\ln 2}{\ln 2 + t(-\ln(1 - p))}\right)^{t + \ln 2 / (-\ln(1 - p))}. \quad (1)$$

p 183, l 5. sum is smaller than the integral, but it is larger than the integral that
is obtained if the curve is shifted right one unit.

p 186. The Bernoulli numbers $B_m$, which occur as coefficients in the Bernoulli polynomials, are defined by the recurrence

$$B_0 = 1, \quad B_n = \frac{-1}{n + 1} \sum_{0 \leq i \leq n - 1} \binom{n + 1}{i} B_i \quad \text{for } n \geq 1.$$  

Except for $B_1$ all the Bernoulli numbers of odd index are zero. The first few
Bernoulli numbers are given in Table 4.1.
Higher-degree approximations can be obtained by integrating eq. (153) by parts. To do this we will need to integrate $B_1(x)$. As the integration by parts proceeds, it will give rise to a sequence of polynomials called Bernoulli polynomials. The Bernoulli polynomial $B_m(x)$ is defined recursively as $\int mB_{m-1}(x)\,dx$ with the constant of integration equal to $B_m$. An equivalent definition is

$$\sum_{1 \leq i < n} f(i) = \sum_{1 \leq i < m} f(i) + \int_{m}^{n} f(x)\,dx - \frac{1}{2}(f(n) - f(m)) + \int_{m}^{n} B_1(\{x\})f'(x)\,dx.$$  

EXERCISES

1. Show that

$$\sum_{1 \leq i < n} i^{1/2} \geq \frac{2}{3} n^{3/2} - \frac{1}{2} n^{1/2} - \frac{241}{1920} + \frac{1}{24} n^{-1/2} - \frac{1}{1920} n^{-5/2}$$

and

$$\sum_{1 \leq i < n} i^{1/2} \leq \frac{2}{3} n^{3/2} - \frac{1}{2} n^{1/2} - \frac{1}{8} + \frac{1}{24} n^{-1/2}.$$  

Hint: Use the generalized Euler summation formula with $m = 3$.

Every lower limit of 0 should be changed to 1 on the summations.

2. Solve

$$T_n = a(n + 1) \log(n + 1) + \frac{b - 3a + c}{2}n + \frac{c - a - b}{2}.$$
The classical algorithm for matrix multiplication (Algorithm 1.9 [modified
to save one addition in the inner loop])

p 217

EXERCISES

1. Consider a sequence of $2^k - 1$ numbers with the middle number first, then the
smaller numbers, and then the larger numbers. The small part (and also the large
part) obey the same rule. For $k = 3$ the binary sequence 100, 001, 011, 010,
101, 111 is such a sequence. Show that the sequence formed by these rules causes
the Split algorithm to produce an equal division each time.

2. Show that the sequence of the last exercise results in Split exiting the first time it
gets to Step 7.

3. Show that if Split exits the first time it gets to Step 7 and if the time to go around
the loop in Step 3 is equal to the time to go around the loop in Step 6, then
the time for Quicksort obeys the recurrence $T_n = an + b + T_{n_1} + T_{n_2}$ when the
splitting element is such that Split divides the data into sets of size $n_1$ and $n_2$
(where $n = n_1 + n_2 + 1$).

Remove the previous exercise 1 and renumber the remaining ones.

The changes on p. 217 result in pages 217–244 being retypeset. Some
thought should be given to ways to reduce the number of reset pages.

p 220, l 9. Since 2 is less than 3,
p 223, l 2. The material
p 226 l 2. $F_{n+1} = (1/\sqrt{5})(\hat\phi^{n+1} - \hat\phi^{n+1}).$
p 235, eq. 186.

\[
\sum_{0 \leq n < k} z^n \sum_{0 \leq i \leq n} a_i T_{n-i} + \sum_{n \geq k} b_n z^n.
\]

p 236, l 6. By replacing $i$ with $k - i$ the left side of eq. (190) can be rewritten as
p 237, l 7 to –8.

\[
\frac{T_p(z)}{(1 - \lambda_p z)^\beta_p} = (-1)^{\beta_p+1} \sum_{0 \leq q \leq \max\{0, q - \beta_p + 1\}} \sum_{i \leq q} c_{p, q-i} \left(\frac{-i - 1}{\beta_p - 1}\right) \lambda_p^{i - q}, \quad (202)
\]

so using $d_q(p)$ for the coefficient of $z^q$ in $T_p(z)/(1 - \lambda_p z)^\beta_p$, we have

\[
d_q(p) = (-1)^{\beta_p+1} \sum_{\max\{0, q - \beta_p + 1\} \leq i \leq q} c_{p, q-i} \left(\frac{-i - 1}{\beta_p - 1}\right) \lambda_p^i, \quad (203)
\]

\[
= (-1)^{\beta_p+1} \lambda_p^q \sum_{\max\{0, q - \beta_p + 1\} \leq i \leq q} c_{p, q-i} \left(\frac{-i - 1}{\beta_p - 1}\right) \lambda_p^{-q+i}. \quad (204)
\]

Now replace $i - q$ by $i'$ ($i = q + i'$) and drop the prime to obtain

\[
d_q(p) = (-1)^{\beta_p+1} \lambda_p^q \sum_{\max\{-q, -\beta_p + 1\} \leq i \leq 0} c_{p, -i} \left(\frac{-i - q - 1}{\beta_p - 1}\right) \lambda_p^i. \quad (205)
\]
Finally replace $i$ by $-i$ to obtain

$$d_q(p) = (-1)^{p+1} \lambda_p^q \sum_{0 \leq i \leq \min(q, p-1)} c_{p, i} \left( \frac{i - q - 1}{\beta_p - 1} \right) \lambda_i^{-p}.$$  \hfill (206)

Thus the form of $d_q(p)$ is an exponential in $q$ (i.e., $\lambda_p^q$) times a polynomial in $q$ of degree $\beta_p - 1$ [i.e., the rest of the right side of eq. (206)].

To obtain the coefficient of $z^q$ in the generating function $G(z)$ for the homogeneous case, it is necessary to sum the contributions from each $T_p(z)$ to obtain

$$\sum_{1 \leq p \leq j} d_q(p).$$  \hfill (207)

$p. 239, eq. 219$

$$f_{1,0} 2^1 + f_{1,1} 1 \cdot 2^1 = 1,$$

$p. 239, l. 5$. so the solution that matches

$p. 240, ex. 2$. Find the general solution of the recurrence $F_{n+2} + F_{n+1} + F_n = n$. Hint: First try to find a particular solution of the form $c_1 n + c_0$.

$p. 240, ex. 4$.

$$T_n = 4T_{n-1} - 5T_{n-2}$$

$p. 247, l. 3$. the grand median and the remaining elements. The remaining

$p. 248, l. 2$. For eq. (253)

$p. 249, eq. (255)$.

$$\beta \geq \frac{1}{3} \beta + \alpha + \beta + \frac{13}{12} \beta + 4\alpha + 4\beta \geq 73\frac{1}{3} + 5\frac{2}{3} \beta. \hfill (256)$$

Solving eq. (256) gives

$$\beta \leq -12\frac{4}{3}. \hfill (257)$$

$p. 249, l. 8$. $\beta = -12\frac{4}{3}$.

$p. 249, l. 10$ give

$p. 249, eq. (259)$.

$$14\frac{2}{3} n - 7830$$

$p. 256, ex. 3$.

$$J_{n+1}(z) - \frac{2n}{z} J_n(z) + J_{n-1}(z) = 0,$$

$p. 256, ex. 4$. Show that the $F(n) = 2F_1(n,(b + a)/(c + 1);b;c + 1)$ is a solution to the recurrence $ncF_{n+1} + [(1-c)n + a]F_n + (b-n)F_{n-1} = 0$. Find the general solution to the recurrence.

$p. 256, ex. 4$. Show that this equation is satisfied by $n^r$. 

$$\sum_{0 \leq i < k+1} r_{i+1}(n)[B(n-i) - B(n-i-1)] = r_0(n)B(n) - r_{k+1}(n)B(n-k-1).$$

$p. 261, ex. 4$. Show that this equation is satisfied by $n^r$. 


$$A_i = (i - 1)! \left( A_1 + Y_0 \sum_{1 \leq j < i} \frac{1}{j!} \right).$$

For large $i$ the sum is close to $e - 1$.

$$a_0(n)B_j(n) = - \sum_{1 \leq i \leq k} a_i(n)B_j(n - i).$$

For large $i$ the sum is close to $e - 1$.

p 266. Exercises move to page 268.

p 266, eq. 100.

$$a_0(n)B_j(n) = - \sum_{1 \leq i \leq k} a_i(n)B_j(n - i).$$

p 268. New section called Annihilation of the Nonhomogeneous Part.

These two changes cause the rest of the chapter (pp 266–276) to be reset.

p 272, l 6. with the boundary conditions $p_{1i} = \delta_{0i}$

p 273, eq. 146.

$$G_n(z) = \left( \frac{-1}{z} \right)^n \left( \frac{-z}{n} \right).$$

p 273, eq. 147.

$$G_n(z) = \frac{(-1)^n}{n!z} \sum_i (-1)^{n-i} \binom{n}{i} (-z)^i = \frac{1}{n!} \sum_i \binom{n}{i} z^{i-1},$$

p 273, l 10 The right side of eq. (143) is the product

p 274, l 17. $P_1$ and $P_2$

p 275 eq. 165.

$$q_{13}i^3 + q_{12}i^2 + q_{11}i + q_{10} = i(q_{13}(i - 1)^3 + q_{12}(i - 1)^2 + q_{11}(i - 1) + q_{10}) + i^2 - 2i,$$

p 275 eq. 166.

$$q_{13}i^3 + q_{12}i^2 + q_{11}i + q_{10} = q_{13}i^4 - 3q_{13}i^3 + 3q_{13}i^2 - q_{13}i + q_{12}i^3 - 2q_{12}i^2$$

$$+ q_{12}i + q_{11}i^2 - q_{11}i + q_{10}i + i^2 - 2i.$$

p 276 eq. 170

$$q_{13} - q_{12} + 2q_{11} - q_{10} = -2$$

p 278, l 10. The boundary condition is $C_0 = 0$.

p 278, eq. 7.

$$C_n = n + 1 + \frac{2}{n} \sum_{0 \leq i < n} C_i \quad \text{for } n \geq 1,$$

p 279, eq. 10.

$$nC_n - (n - 1)C_{n-1} = n^2 - (n - 1)^2 + n - (n - 1)$$

$$+ 2 \sum_{0 \leq i < n} C_i - 2 \sum_{0 \leq i < n-1} C_i$$

$$= 2n + 2C_{n-1} \quad \text{for } n \geq 2,$$
a first order linear equation. The solution for the boundary condition $C_1 = 2$ is

$$C_n = 2 \prod_{2 \leq j \leq n} \frac{j + 1}{j} + \sum_{2 \leq i \leq n} 2 \prod_{i < j \leq n} \frac{j + 1}{j}$$

$$= 2 \sum_{1 \leq i \leq n} \frac{n + 1}{i + 1} = 2(n + 1)(H_{n+1} - 1).$$

To simplify this equation, we can use the fact that $C(x)$ obeys eq. (159). We can rewrite eq. (159) as

$$\sum_{0 \leq i \leq n-1} C_i C_{n-i} = C_{n+1} - C_n.$$
\[ f_n(x) = \min_{0 \leq y \leq x} \{ y \ln y + f_{n-1}(x-y) \} \]

p 315, -9 label any of the leaves in its left subtree.

p 322, eq. 204.

\[ t_n = n - 1 + \sum_{1 \leq i \leq n-1} (t_i + t_{n-i}), \quad \text{204} \]

p 331, Step 4. If \( i > F_{2t+1} \) and \( Y_i = \text{true} \), then set \( M_{i-F_{2t+1}} \leftarrow \text{true} \).

p 339, eq. 70
\[ 1 = c_{11} + c_{12} + c_{13} \]

p 339, eq. 73
\[ c_{11} = \frac{2 - \lambda_2 - \lambda_3 + \lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \]

p 339, eq. 77
\[ c_{11} = \frac{\lambda^2}{(3\lambda_1 + 1)(\lambda_1 - 1)} \]

p 340, eq. 85
\[ = \frac{(\lambda + 1)\lambda^2}{3\lambda + 1}\lambda^n + O(0.738^n) \]

p 340, eq. 91
\[ x_3(l) = \frac{\lambda^{l+1}}{(3\lambda + 1)(\lambda - 1)} + O(0.738^l) \approx 0.336\lambda^l \]

p 341, l 16. where \( I \) is the identity matrix and \( A \) is the matrix

p 342, eqs. 104–106.
\[ T_{\text{Polyphase Merge}} = \sum_{1 \leq j \leq n} \frac{L\lambda^2}{(3\lambda + 1)(\lambda - 1)} + O(0.401^l) \]
\[ = \frac{nL\lambda^2}{(3\lambda + 1)(\lambda - 1)} + O(1) \]
\[ = \frac{\lambda^2}{(3\lambda + 1)(\lambda - 1)\log N} + O(1) \approx 0.704\lambda \log N. \]

p 342, l 14. Merge about 80 percent faster than Simple Merge

p 343. Check with Q. Stout to see if there are any more errors on this page.

p 344, eq. 114.
\[ \frac{dP_k(t)}{dt} = \mu P_{k-1}(t) - (k\lambda + \mu)P_k(t) + (k + 1)\lambda P_{k+1}(t), \]

p 345, eq. 117.
\[ A(t, v) = at + 2 \sum_i \binom{t}{i} p^i (1-p)^{t-i} A(t-i, v-1). \]

p 346, l 11. (using the boundary conditions of the original problem), which reduces to
p 346, -2. The general solution of eq. (126) is the function
\[ x'(n, i) = x \left( \frac{sn - qi}{ps - qr} - \frac{rn + pi}{ps - qr} \right) = x(ci + a[n - i], di + b[n - i]), \]
\[ y'(n, i) = y \left( \frac{sn - qi}{ps - qr} - \frac{rn + pi}{ps - qr} \right) = y(ci + a[n - i], di + b[n - i]), \]
\[ z'(n, i) = z \left( \frac{sn - qi}{ps - qr} - \frac{rn + pi}{ps - qr} \right) = z(ci + a[n - i], di + b[n - i]). \]

\[ a_{ni} = \left( \left\lfloor \frac{n+i}{2} \right\rfloor \right) + \left( \left\lfloor \frac{n+i-1}{2} \right\rfloor \right). \]

When \( n = 1 \), \( H_{ni} \) is zero except for \( i = 0 \), and \( H_{10} = 1 \), so \( F \) obeys

\[ \frac{i}{n} T_{ni} = (n - i)T_{n-1,i-1} + iT_{n-1,i} \quad \text{for } n \geq 3. \]

The number of \( k \)-dimensional cubes (\( k \)-cubes) in an \( n \)-cube obeys the recurrence

\[ |T_{9,1}|. \]

TABLE 9.1. The nontrivial equivalence classes for the flow graph in Figure 9.5. Two nodes are equivalent if there exists a node with arcs to both of them. The equivalence classes with just one node are not given.

<table>
<thead>
<tr>
<th>Parent Class</th>
<th>Parent Class</th>
<th>Parent Class</th>
<th>Parent Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4, 5</td>
<td>11</td>
<td>11, 12</td>
</tr>
<tr>
<td>5</td>
<td>3, 6</td>
<td>14</td>
<td>15, 16</td>
</tr>
<tr>
<td>6</td>
<td>2, 7</td>
<td>17</td>
<td>17, 18</td>
</tr>
<tr>
<td>8</td>
<td>9, 14</td>
<td>18</td>
<td>19, 21</td>
</tr>
<tr>
<td>9</td>
<td>10, 13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 9.9. 2 2

\[ nt_0 + t_1 f(A, S) \]

This is a much better

If \( y \neq t \), then set \( z \leftarrow y \), \( y \leftarrow \text{parent}(y) \), \( \text{parent}(z) \leftarrow t \), and repeat this step.

A dot is needed in the middle of the three long paths. That is in the right path of the third figure, the second from the left path of the fourth figure and the lower part of the rightmost path of the fourth figure.

\[ 2^{A_{r-1}} \]

to evaluating the polynomial \( \sum_{0 \leq k < n} x_k y^k \) at the points \( y_j = \omega^j \) for \( 0 \leq j < n. \)
p 394, l 14 and many other
p 395, l 11. so eq. (37)
p 397, l –2 one for each of the two subpart,
p 398.

<table>
<thead>
<tr>
<th>Level</th>
<th>Calculation</th>
</tr>
</thead>
</table>
| 2     | \( y_0 \leftarrow y_0' + y_1' = x_j + x_{j+4} \)  
       | \( y_1 \leftarrow y_1' - y_1' = x_j - x_{j+4} \)  |
| 1     | \( y_0 \leftarrow y_0' + y_2' = x_j + x_{j+4} + x_{j+2} + x_{j+6} \)  
       | \( y_2 \leftarrow y_0' - y_2' = x_j + x_{j+4} - x_{j+2} - x_{j+6} \)  
       | \( y_1 \leftarrow y_1' + \omega^2 y_3' = x_j - x_{j+4} + \omega^2 x_{j+2} - \omega^2 x_{j+6} \)  
       | \( y_3 \leftarrow y_1' - \omega^2 y_3' = x_j - x_{j+4} - \omega^2 x_{j+2} + \omega^2 x_{j+6} \)  |
| 0     | \( y_0 \leftarrow y_0' + y_4' = x_0 + x_4 + x_2 + x_6 \)  
       | \( y_4 \leftarrow y_0' - y_4' = x_0 + x_4 + x_2 + x_6 \)  
       | \( y_1 \leftarrow y_1' + \omega y_5' = x_0 - x_4 + \omega x_2 - \omega x_6 \)  
       | \( y_5 \leftarrow y_1' - \omega y_5' = x_0 - x_4 + \omega x_2 - \omega x_6 \)  
       | \( y_2 \leftarrow y_2' + \omega^2 y_6' = x_0 + x_4 - x_2 - x_6 \)  
       | \( y_6 \leftarrow y_2' - \omega^2 y_6' = x_0 + x_4 - x_2 - x_6 \)  
       | \( y_3 \leftarrow y_3' + \omega^3 y_7' = x_0 - x_4 - \omega x_2 + \omega x_6 \)  
       | \( y_7 \leftarrow y_3' - \omega^3 y_7' = x_0 - x_4 - \omega x_2 + \omega x_6 \)  |

p 399, l 12. different ways and then does not make any additional use of the original
p 399, l 112. the variable \( q \) used in Step 2 has a leading zero, followed by the \( j \)
higher-order bits of the \( i \) in reversed order, followed by \( k - j - 1 \) trailing zeros.
p 399, Step 2. For \( 0 \leq i < n \) do the rest of this step. If \( i_{k-j-1} = 0 \), then set
\( q \leftarrow 0i_{k-j}i_{k-j+1} \ldots i_{k-100} \ldots 0 \), odd \( \leftarrow \omega^q x_{i+2k-j-1} \),
p 401, l 8. We need (see exercise 3.4.1–2)
p 405, l 13. (for special values of \( i \) such numbers are called Mersenne primes in the
first case and Fermat primes in the second case).
p 420, l 14.

Arrays \( T_1, T_2, T_3, T_4 \), and \( T_5 \).
p 454 eq. 32

\[
F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{(y-\mu)^2}{2\sigma^2} \right) dy.
\]
\[ f(x) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right). \]

p 463 eq. 70.
\[ \Pr(X \geq 70) = \left( \frac{1}{2} \right)^{100} \sum_{i \geq 70} \binom{100}{i} \approx 3.98 \times 10^{-5} \]

p 463 eq. 73.
\[ \Pr(|X - 50| \geq 20) \leq \frac{25}{400} \approx 0.062 \] (2)

p 488, eq. 7
\[ \int_{\Gamma} \frac{dz}{z - z_0} = \int_0^1 e^{-2\pi i \theta} d(e^{2\pi i \theta}) = 2\pi i \int_0^1 d\theta = 2\pi i. \] (7)

p 494, eq. 25
\[ f(x) = x^{m-n} \left( \frac{1}{1-x_1/x} \right) \left( \frac{1}{1-x_2/x} \right) \cdots \left( \frac{1}{1-x_n/x} \right) = x^{m-n} + (x_1 + x_2 + \cdots + x_n)x^{m-n-1} + \cdots, \] (25)