Assignment 4: Residues, NFAs

Solved practice problems are numbered in red,
assigned problems and sub-problems in green.

1. (30%)

   (a) Let $L_{a1} \subseteq \{\mathtt{a}, \mathtt{b}\}^*$ consist of the strings with $\mathtt{a}$ in ***some*** odd position. Identify the residues of L and build a partial-automaton M_{a1} from them.

   (b) Let $L_{b0} \subseteq \{\mathtt{a}, \mathtt{b}\}^*$ consist of the strings with $\mathtt{b}$ in ***all*** even positions. Identify the residues of L and build a partial-automaton M_{b0} from them.

   (c) What needs to be changed in the definition of product automata to obtain a correct definition of the product of ***partial-***automata?

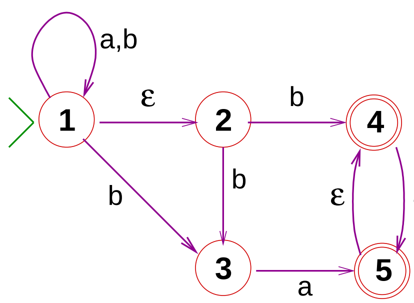
   (d) Using your answer to (c) construct the conjunctive product of M_{a1} and M_{b0} to obtain a partial-automaton that recognizes $L_{a1} \cap L_{b0}$.

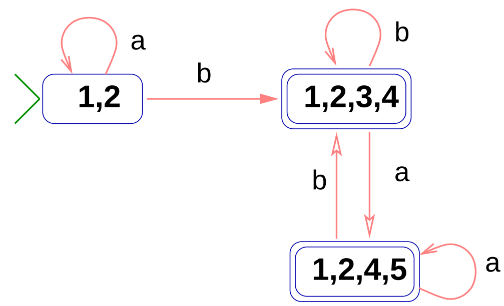
2. (20%) A CFA (conjunctive NFA) C (over alphabet Σ) is like an NFA, except that a string w is accepted if *every* state p such that $s \xrightarrow{w} p$ is accepting.

   (a) Prove that a language is recognized by a CFA iff its complement is recognized by an NFA.

   (b) Conclude that every language recognized by a CFA is recognized by a DFA.

A. Convert the following NFA into an equivalent DFA.

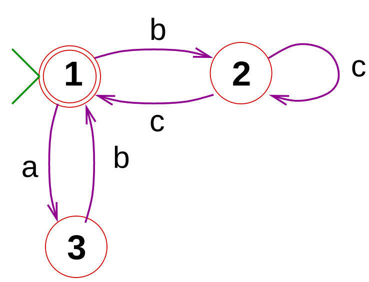




3. (15%)

i. Construct an NFA N with three states that recognizes the language $L = \mathcal{L}((\mathtt{ab} \cup \mathtt{bc}^+)^*)$.

Solution.



(a) Convert the NFA N above into an equivalent DFA. (Place any sink in the center of your diagram.)

4. (20%) Use ***residues*** to show that $L \subseteq \{\mathtt{a}, \mathtt{b}\}^*$ defined by $L = \{x \cdot \mathtt{a}^n \mid n > 0, \ |x| = n\}$ is not regular.

5. (15%) Convert the following NFA into an equivalent DFA.

(Note that two states here have the same residue, so this is not a minimal DFA for L.)