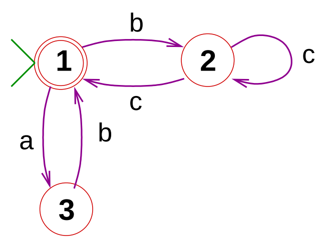
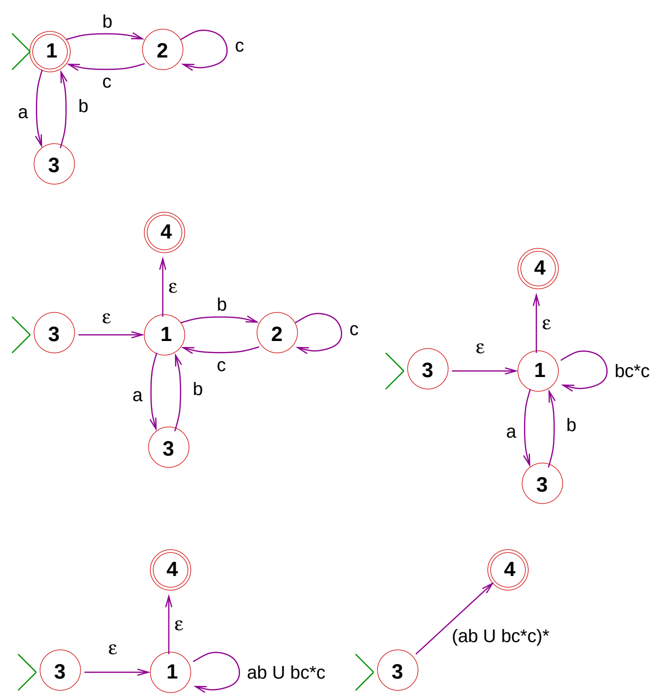
Assignment 5: Regular languages, other machines

1. (30%) Convert each of the following NFA's N into an equivalent regular expression α (i.e. such that $\mathcal{L}(\alpha) = \mathcal{L}(N)$). Exhibit the stages of each conversion.

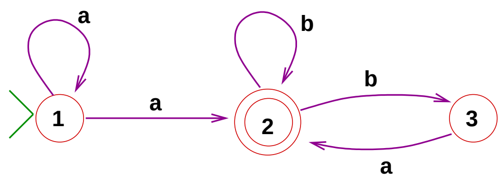
i.



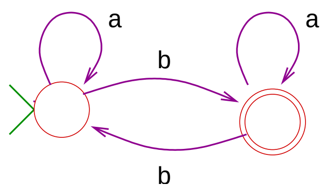
Solution.



(a)



(b)



2. (25%)

(a) Construct an LBA that recognizes the language
$L = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^{n+m} \mid n, m \geqslant 1\}$.

(b) (25%) Give the computation-trace for **abcc**.
assdsub Give the computation-trace for **abaccc**.

3. (25%)

(a) Construct an LBA recognizing $L = \{w \cdot w^R \mid w \in \{\mathtt{a}, \mathtt{b}\}^* \}$, where w^R is the reverse of w. Define your LBA in a modular format. [Hint: This is similar to the problem of accepting the strings $\mathtt{a}^n\mathtt{b}^n$ considered in class.]

(b) Give the computation trace of your acceptor for **abba**.

A. **i.** Construct a Turing transducer that replaces the last input symbol by ba.

Solution. Start state S, print state P.

$S \xrightarrow{\sigma\,(+)} S \quad (\sigma \neq \sqcup)$

$S \xrightarrow{\sqcup\,(\mathtt{a})} R$

$R \xrightarrow{\mathtt{a}\,(-)} B$

$B \xrightarrow{\sigma\,(\mathtt{b})} P$

ii. Give the computation trace of your transducer for input baa.

$$
\begin{aligned}
(S, \underline{>}\texttt{baa}\sqcup) &\Rightarrow (S, >\underline{\texttt{b}}\texttt{aa}\sqcup) \\
&\Rightarrow (S, >\texttt{b}\underline{\texttt{a}}\texttt{a}\sqcup) \\
&\Rightarrow (S, >\texttt{ba}\underline{\texttt{a}}\sqcup) \\
&\Rightarrow (S, >\texttt{baa}\underline{\sqcup}) \\
&\Rightarrow (R, >\texttt{baa}\underline{\texttt{a}}) \\
&\Rightarrow (B, >\texttt{ba}\underline{\texttt{a}}\texttt{a}) \\
&\Rightarrow (P, >\texttt{ba}\underline{\texttt{b}}\texttt{a})
\end{aligned}
$$

4. (20%) Construct a Turing transducer that swaps the first and last input symbols. For example, **abcd** is mapped to **dbca**. (Single-letter strings and ε are mapped to themselves.)