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This is a presentation of Relation-Oriented Programming. In order to appreciate Logic-Oriented Programming, we must first understand relations. We use the term relation in the following way. A function is a mapping from a value to another value, which can be modelled by a set of pairs where no two pairs have the same first value. A relation can be modelled by a set of pairs without this restriction.

We introduce more of the Kanren logic system, which we first encountered while examining Outcome-Oriented Programming (OOP).

This paper addresses the notion of relation in two separate ways. First, we show how using lambda and outcomes is powerful enough to express Relation-Oriented Programming in its entirety. Then, we show that by a careful crafting of some macros, we can make Relation-Oriented Programming seem more natural.

We will use the following family-oriented relations throughout: father, mother, child, grandparent, ancestor, common-ancestor, younger-common-ancestor, and youngest-common-ancestor. If we are going to have a program that responds to these relations, we need some real data: Jon is the father of Sam and Hal; Sam is the father of Rob, Jay, and Roz; Hal is the father of Ted; and Rob is the father of Sal and Pat. Figure 1 shows two different representations of this paternal family tree.

We can represent this paternal relationship with the following father procedure.

(define father
  (lambda (dad child)
    (any
      (all!! (== dad 'jon) (== child 'sam))
      (all!! (== dad 'jon) (== child 'hal))
    ))

1 Consider the square function defined over integral values. We can represent this function as the following infinite set of pairs,

\{ (-3 9) (-2 4) (-1 1) (0 0) (1 1) (2 4) (3 9) ... \}

The left-hand-side of a pair denotes the input to the square function, while the right-hand-side of the pair represents the value returned by the function.

2 For example, we can represent the square-root relation over the domain of non-negative integers as follows,

\{ (0 0) (0 -0) (1 1) (1 -1) (2 √2) (2 -√2) (3 √3) (3 -√3) (4 2) (4 -2) ... \}

Note that square-root is a relation but not a function, since each input value (e.g., 4) is associated with two output values (2 and -2, in this case).
There are eight \( \text{(all!! ...) \dots} \) expressions in the \texttt{father} procedure, which we can reorder in any of \( 8! \) different ways. Regardless of the order we choose for the \( \text{(all!! ...) \dots} \) expressions, our \texttt{father} procedure will impose that same ordering upon the children occupying a given level of the family tree. In the \texttt{father} procedure given above, Sam comes “before” Hal, even though both Sam and Hal are children of Jon. This ordering forces us to choose the left-hand tree in Figure 1 as our preferred graphical representation of the family tree, since a breadth-first left-to-right traversal of this tree will yield Jon’s descendents in the order specified in the \texttt{father} procedure.

If we were to switch the order of the \( \text{(all!! ...) \dots} \) expressions in the \texttt{father} procedure, the order of the answers produced by our Relation-Oriented programs might change as well, along with the amount of computational resources consumed by those programs.

We have one special form to add to those introduced in the Outcome-Oriented Programming paper. Its behavior is identical to \texttt{exists} and it is called \texttt{let-lv}. When we build an antecedent for \texttt{query}, we use it to introduce logically-scoped variables.

Each time \texttt{trace-vars} is invoked in the code below, it displays the content of a portion (it may be all of it) of a substitution. After each \texttt{trace-vars} completes, it passes the current substitution to \texttt{fail}. That causes the computation to back up.
through the trace-vars and then through the father. Backing into it, will cause the logic variables f and c to get rebound and then that substitution is sent to trace-vars, etc. Since the final outcome is failure, we know that the result will be the empty list, indicating failure.

> (query
  (let-lv (f c)
    (all
      (father f c)
      (trace-vars ':: (f c))
      fail)))))
:: (f c) (jon sam)
:: (f c) (jon hal)
:: (f c) (sam rob)
:: (f c) (sam jay)
:: (f c) (sam roz)
:: (f c) (hal ted)
:: (f c) (rob sal)
:: (f c) (rob pat)
()

If the number of answers is relatively small, then a simple macro like the one below meets our needs. As is evident, the only things that we abstracted over were the list of variables (f c) and the antecedent expression, (father f c). Why must this be a macro? Because the id ... in the let-lv expression introduce lexical scope.

(define-syntax answers
  (syntax-rules ()
    [(_ (id ...) ant-exp)
      (query
        (let-lv (id ...)
          (all ant-exp (trace-vars ':: (id ...)) fail)))])
))

> (answers (f c) (father f c))
:: (f c) (jon sam)
:: (f c) (jon hal)
:: (f c) (sam rob)
:: (f c) (sam jay)
:: (f c) (sam roz)
:: (f c) (hal ted)
:: (f c) (rob sal)
:: (f c) (rob pat)
()

We should not always use the answers macro, of course, since it is possible to give an antecedent expression that produces an unbounded number of answers. In Logic-Oriented-Programming, we see one way to control that.
We can take projections of this relation with examples like this:

```lisp
> (answers (c) (father 'sam c))
:: (c) (rob)
:: (c) (jay)
:: (c) (roz)
()
> (answers (f) (father f 'rob))
:: (f) (sam)
()
```

The first example yields three substitutions, each of which gives a child of Sam. The second example yields a single substitution, since Rob has only one father.

We can define \texttt{child} by simply claiming that in our world,

\begin{verbatim}
(define child
  (lambda (child dad)
    (father dad child)))
\end{verbatim}

```lisp
> (answers (c f) (child c f))
:: (c f) (sam jon)
:: (c f) (hal jon)
:: (c f) (rob sam)
:: (c f) (jay sam)
:: (c f) (roz sam)
:: (c f) (ted hal)
:: (c f) (sal rob)
:: (c f) (pat rob)
()
```

Thus, to show that \texttt{c} is the child of \texttt{f}, we need only show that \texttt{f} is the father of \texttt{c}. Of course, this assumes that in our population, no woman has a child. If that were not the case, then we would likely define \texttt{child} this way.

```lisp
(define child
  (lambda (child parent)
    (any
      (father parent child)
      (mother parent child))))
```
Exercise 1: Create a new relation mother and verify that this revised definition of child makes sense. What happens when the same child is in both father and mother? 

We next define the grandparent relation, keeping in mind that a grandparent is the parent of the parent of a grandchild.

\[
\text{(define grandparent} \\
\quad \text{(lambda (gr-parent gr-child)} \\
\quad \quad \text{(exists (parent)} \\
\quad \quad \quad \text{(all} \\
\quad \quad \quad \quad \text{(child parent gr-parent)} \\
\quad \quad \quad \quad \text{(child gr-child parent)))))}
\]

\[
> \text{(answers (gp gc) (grandparent gp gc))} \\
\quad :- \text{(gp gc) (jon rob)} \\
\quad :- \text{(gp gc) (jon jay)} \\
\quad :- \text{(gp gc) (jon roz)} \\
\quad :- \text{(gp gc) (jon ted)} \\
\quad :- \text{(gp gc) (sam sal)} \\
\quad :- \text{(gp gc) (sam pat)} \\
\quad ()
\]

The relation ancestor determines if a person is a parent, grandparent, greatgrandparent, greatgreatgrandparent, etc. of someone.

\[
\text{(define ancestor} \\
\quad \text{(lambda (anc desc)} \\
\quad \quad \text{(any} \\
\quad \quad \quad \text{(child desc anc)} \\
\quad \quad \quad \text{(exists (parent)} \\
\quad \quad \quad \quad \text{(all} \\
\quad \quad \quad \quad \quad \text{(child desc parent)} \\
\quad \quad \quad \quad \quad \text{(ancestor anc parent)))))}
\]

\[
> \text{(answers (a d) (ancestor a d))} \\
\quad :- \text{(a d) (jon sam)} \\
\quad :- \text{(a d) (jon hal)} \\
\quad :- \text{(a d) (sam rob)} \\
\quad :- \text{(a d) (sam jay)} \\
\quad :- \text{(a d) (sam roz)} \\
\quad :- \text{(a d) (hal ted)} \\
\quad :- \text{(a d) (rob sal)} \\
\quad :- \text{(a d) (rob pat)} \\
\quad :- \text{(a d) (jon rob)} \\
\quad :- \text{(a d) (jon jay)} \\
\quad :- \text{(a d) (jon roz)}
\]
If we look at the output of this example, we can see that a person might have multiple ancestors. For example, Pat has three ancestors. We might also want a relation that determines if two people share a common ancestor. For example, Jon is the common ancestor of everyone but himself.

```
(define common-ancestor
  (lambda (anc desc1 desc2)
    (all
     (ancestor anc desc1)
     (ancestor anc desc2)
     (project (desc1 desc2)
         (predicate (not (eq? desc1 desc2)))))))
```

Exercise 2: How many answers would there be if the test had been `(answers (a d1 d2) (common-ancestor a d1 d2))`? If we removed the last antecedent in the definition of `common-ancestor`, how many answers would we get? ◦

We might want to know of any two common ancestors, which one is younger. For example, Jon is clearly the older of Sam and Jon.

```
(define younger-common-ancestor
  (lambda (young-anc old-anc desc1 desc2)
    (all
     (common-ancestor young-anc desc1 desc2)
     (common-ancestor old-anc desc1 desc2)
     (ancestor old-anc young-anc))))
```

Exercise 2: How many answers would there be if the test had been `(answers (a d1 d2) (common-ancestor a d1 d2))`? If we removed the last antecedent in the definition of `common-ancestor`, how many answers would we get? ◦

We might want to know given two people, who their youngest common ancestor is. For example, let us determine the youngest common ancestor of Jay and Pat.
Tutorial

(define youngest-common-ancestor
  (lambda (young-anc desc1 desc2)
    (all
      (common-ancestor young-anc desc1 desc2)
      (fails
        (exists (y)
          (younger-common-ancestor y young-anc desc1 desc2))))))

> (answers (y) (youngest-common-ancestor y 'jay 'pat))
:: (y) (sam)
()

We discover that Sam, who is Jay’s father and Pat’s grandfather, is the youngest common ancestor of Jay and Pat.

What we have just demonstrated is that conventional logic programming can be managed at the level of writing Scheme functions and using outcomes. Next, we show how we can change the perspective just a little and yield equivalent programs. To do this, we add the following rule to the grammar introduced in Outcome-Oriented Programming.

R :: (relation (Id*) (to-show Term*) A)
  | (fact (Id*) Term*)
  | (extend-relation Arity R*)
  | (intersect-relation Arity R*)

Arity :: (Id*)

The relation macro is similar to lambda in that it introduces lexical scope. Here is a relation, ancestor-of-both, that uses the relation macro.

3 The length of the Arity list must match the number of terms in the relation or relations being extended or intersected. For example, the father relation expresses the relationship between two terms, the father and the child. When extending the father relation, the Arity list must therefore contain exactly two identifiers.

Only the length of the Arity list is significant, and the actual identifiers appearing in the list may be chosen arbitrarily. The following expressions are therefore equivalent, since each expression’s Arity list is of length two:

(extend-relation (a1 a2) father)
(extend-relation (dad child) father)
(extend-relation (foo bar) father).
(define ancestor-of-both
  (relation (a d1 d2)
    (to-show a '(',d1 ,d2))
    (all
      (ancestor a d1)
      (ancestor a d2))))

> (answers (a) (ancestor-of-both a '(pat jay)))
  :: (a) (sam)
  :: (a) (jon)
  ()

> (answers (d1 d2) (ancestor-of-both 'sam '(',d1 ,d2)))
  :: (d1 d2) (rob rob)
  :: (d1 d2) (rob jay)
  :: (d1 d2) (rob roz)
  :: (d1 d2) (rob sal)
  :: (d1 d2) (rob pat)
  :: (d1 d2) (jay rob)
  :: (d1 d2) (jay jay)
  :: (d1 d2) (jay roz)
  :: (d1 d2) (jay sal)
  :: (d1 d2) (jay pat)
  :: (d1 d2) (roz rob)
  :: (d1 d2) (roz jay)
  :: (d1 d2) (roz roz)
  :: (d1 d2) (roz sal)
  :: (d1 d2) (roz pat)
  :: (d1 d2) (sal rob)
  :: (d1 d2) (sal jay)
  :: (d1 d2) (sal roz)
  :: (d1 d2) (sal sal)
  :: (d1 d2) (sal pat)
  :: (d1 d2) (pat rob)
  :: (d1 d2) (pat jay)
  :: (d1 d2) (pat roz)
  :: (d1 d2) (pat sal)
  :: (d1 d2) (pat pat)
  ()

Below we show how ancestor-of-both would have been written if we did not have relation. First, the number of arguments after the to-show keyword is two, so we have two lambda formals, with arbitrary names: alpha and beta. Second, we have a list of variables wrapped in an exists expression. This corresponds to the number of distinct variables in the to-show. We can see that there are three variables, a, d1, and d2. Then, each of the introduced variables is placed in an
== expression, one for each of the arguments of the to-show. Finally, the inner (all ...) expression is the same as the one antecedent that follows the to-show expression.

(define ancestor-of-both
  (lambda (alpha beta)
    (exists (a d1 d2)
      (all
        (== alpha a)
        (== beta '(',d1 ,d2))
        (all
          (ancestor a d1)
          (ancestor a d2)))))

There is a special case of relation where there is no antecedent. For example, we might have the fact that Jon is the father of Sam.

(define father-sam
  (fact () 'jon 'sam))

or

(define father-sam
  (relation ()
    (to-show 'jon 'sam)
    succeed))

The list that follows the symbol fact is the list of all the variables used in the fact. Of course, here there are none. Then we could define father using a bunch of facts and a new feature: extend-relation.

(define father
  (extend-relation (f c)
    (fact () 'jon 'sam)
    (fact () 'jon 'hal)
    (fact () 'sam 'rob)
    (fact () 'sam 'jay)
    (fact () 'sam 'roz)
    (fact () 'hal 'ted)
    (fact () 'rob 'sal)
    (fact () 'rob 'pat)))

The first list (f c) is required and it tells extend-relation how many arguments each relation takes. extend-relation acts like a call-by-value function, but since the list of variables (f c) is not evaluated, it is implemented using a macro. The reason extend-relation evaluates all but its first argument is to allow for incremental-relation building.
\(\text{define father} \)
\[(\text{extend-relation} (f \ c))\]
\[
\begin{align*}
\text{father} & \\
(\text{fact} () \ '\text{j}on \ '\text{hal}) & \\
(\text{fact} () \ '\text{sam} \ '\text{rob})) & \\
\end{align*}
\]
which would otherwise loop indefinitely when it is called.

Exercise 3: Implement \textit{ancestors-of-all}, a variation of \textit{ancestor-of-both}, that works with an arbitrarily long list. That would mean that we might want to use \(\{,d \ . \ ,d^*\}\), where \(d\) would be the first descendant and \(d^*\) would be all the other descendants. Solve the problem with and without \textit{relation}. \(\Diamond\)

We could have built our \textit{father} relation in another way.

\(\text{define father} \)
\[(\text{let loop ([facts (list}
\begin{align*}
(\text{fact} () \ '\text{jon} \ '\text{sam}) & \\
(\text{fact} () \ '\text{jon} \ '\text{hal}) & \\
(\text{fact} () \ '\text{sam} \ '\text{rob}) & \\
(\text{fact} () \ '\text{sam} \ '\text{jay}) & \\
(\text{fact} () \ '\text{sam} \ '\text{roz}) & \\
(\text{fact} () \ '\text{hal} \ '\text{ted}) & \\
(\text{fact} () \ '\text{rob} \ '\text{sal}) & \\
(\text{fact} () \ '\text{rob} \ '\text{pat})))) & \\
\end{align*}
\[
\begin{align*}
(\text{cond} & \\
[(\text{null? facts) fail}] & \\
[\text{else (extend-relation} (f \ c) \ (\text{car facts}) \ (\text{loop (cdr facts))})])) & \\
\end{align*}
\]
This follows, since relations, including facts, are first-class values.

We can redefine \textit{parent} using \textit{extend-relation}, and then we can use it to redefine \textit{child}.

\(\text{define parent} \)
\[(\text{extend-relation} (p \ c))\]
\[
\begin{align*}
\text{father} & \\
\text{mother} & \\
\end{align*}
\]
\(\text{define child} \)
\[(\text{relation} (\text{child} \ \text{ther}^4))\]
\[
\begin{align*}
(\text{to-show child} \ \text{ther}) & \\
(\text{parent the\text{r} child}) & \\
\end{align*}
\]
Revising \textit{grandparent} to use \textit{relation} yields

\(^4\) We use the variable \textit{ther} to represent a parent (either a \textit{father} or a \textit{mother}).
(define grandparent
  (relation (gr-parent gr-child)
    (to-show gr-parent gr-child)
    (exists (parent)
      (all
        (child parent gr-parent)
        (child gr-child parent)))))

Revising ancestor gives us

(define ancestor
  (relation (anc desc)
    (to-show anc desc)
    (any
      (child desc anc)
      (exists (parent)
        (all
          (child desc parent)
          (ancestor anc parent))))))

Exercise 4: Implement youngest-common-ancestor using relation.

We next introduce intersect-relation, which acts like extend-relation, but instead of using any, it uses all to build up the relation. Consider the following situation.

(define scouts
  (extend-relation (s)
    (fact () 'rob)
    (fact () 'sue)
    (fact () 'sal)))

(define athletes
  (extend-relation (a)
    (fact () 'roz)
    (fact () 'sue)
    (fact () 'sal)))

(define busy-children
  (intersect-relation (c)
    scouts
    athletes))

(define social-children
  (extend-relation (c)
    scouts
    athletes))
> (answers (c) (social-children c))
  :: (c) (rob)
  :: (c) (sue)
  :: (c) (sal)
  :: (c) (roz)
  :: (c) (sue)
  :: (c) (sal)
  ()

> (answers (c) (busy-children c))
  :: (c) (sue)
  :: (c) (sal)
  ()

When we evaluate (answers (c) (social-children c)) we are presented with the answers sue and sal twice. A slightly different implementation of extend-relation could filter out these duplicate answers.

The advantages of (relation Id* (to-show Term*) A) can hardly be observed using these simple programs. In Logic-Oriented-Programming, we shall see how relation improves the readability of our definitions.