C241 Homework Assignment 1

1. (Exercise 1.1-1) List the following sets:

   (a) \{2^i \mid i \in \mathbb{N} \text{ and } 0 \leq i \leq 8\}

   (b) \{i^2 \mid i \in \mathbb{N} \text{ and } 0 \leq i \leq 8\}

   (c) \{2k + 1 \mid k \in \mathbb{N}\}

   (d) \{m \mid 23 < m < 29 \text{ and } m \text{ is a prime number}\}
2. (Exercise 1.1-2) Let $A = \{a, b\}$; let $B = \{1, 2, 3\}$; let $C = \emptyset$; and let $D = \{a, b, c, d\}$. List the following sets:

(a) $A \cup B$  
(b) $A \cap B$  
(c) $A \times B$  
(d) $\mathcal{P}(A)$  
(e) $B \times \emptyset$  
(f) $A \cup C$  
(g) $A \cap D$  
(h) $A^3$  
(i) $\mathcal{P}(\emptyset)$  
(j) $(D \cap A) \times B$
3. Let $A = \{a, b\}$, $B = \{1, 3, 5\}$, and $C = \{\oplus, \otimes\}$.

(a) List the set $A \times B$.

(b) List the set $A \times (B \times C)$.

(c) List the set $A \times B \times C$. 
4. (Exercise 1.1-3) Let $A = \{a, b\}$; let $B = \{1, 2, 3\}$; and let $E = A \times B$. List the following sets:

(a) $\{(x, y, y) \mid (x, y) \in E\}$

(b) $\{(x, x) \mid x \in E\}$

(c) $\{(y, z) \mid (x, y) \in E \text{ and } z \in B\}$
5. List the first twelve elements of the set

\[ F = \{ f_k \in \mathbb{N} \mid f_1 = 1, \ f_2 = 2, \ \text{and for any } k \geq 1, \ f_{k+2} = f_k + f_{k+1} \} \]

That is, list \( \{f_1, f_2, \ldots, f_{12}\} \)
6. Let $V = \{1, 2, 3\}$. List the set $\{xyxz \mid x, y, z \in V\}$ of words in $V^+$. 
7. (Exercise 1.3-1) Let $V = \{a, b, \$\}$. For each of the following languages $L_i \subseteq V^+$, list enough elements to make it clear what each contains.

(a) In language $L_1$ each word has exactly one $\$ and equally many $a$s as $b$s.

(b) In each word of language $L_2$, $a$s and $b$s alternate with any number of $\$s mixed in.

(c) In each word of language $L_3$, no $a$ occurs next to a $b$.

(d) $L_4 = \{u\$^*v \mid u \in \{a\}^+ \text{ and } v \in \{$, $b\}^+\}$

(e) $L_5 = \{a^{k^*}\$ b^k \mid k \in \mathbb{N}\}$
8. What does this \texttt{stmt} program compute? Trace its execution by hand for a few small values or \( A \) and \( B \). Place a statement in the empty assertion at the end of the program saying what condition holds at that point. Explain why the program satisfies your assertion when it reaches the \texttt{end}.

\[
\{ x = A \in \mathbb{W} \text{ and } y = B \in \mathbb{W} \} \\
\texttt{begin} \\
\texttt{while } x \neq y \texttt{ do} \\
\quad \texttt{if } x < y \\
\quad \quad \texttt{then } y := y - x \\
\quad \quad \texttt{else } x := x - y \\
\texttt{end} \\
\{ \}
\]

(Optional) Write and test a program in the language of your choice (preferably Scheme) that performs the same computation as the given program above.
SUPPLEMENTAL PROBLEM. A small island is ruled by a benevolent queen. One morning, a proclamation is posted in the central common:

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-To All My Subjects---

It has come to my attention that one or more husbands in my realm are unfaithful to their wives. I hereby decree that any woman who learns her husband is unfaithful must shoot him at the stroke of midnight.

Her Highness, The QUEEN

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It’s a small island. Every wife knows who all the unfaithful husbands are, but does not know whether or not her own husband is faithful.

On the third night, shots ring out. How many husbands are/were unfaithful?