Exercise 3.5-1. *Draw two distinct ROBDDs for the term*

$$d \oplus e \oplus (a\,b + a\,c + b\,c)$$

where '$\oplus$' stands for the exclusive-or *operation,*

$$x \oplus y = \overline{x}y + x\overline{y}$$

SOLUTION

HINT. The ROBDDs below are developed from the top down. The 0-branches are labeled with a black dot. The *residual* expressions within the tree show the result of simplifying the target expression with variable $x = 0$ on the $\bullet$ branch and $x = 1$ on the $\circ$ branch.

For example, consider the initial target expression $d \oplus e \oplus (a\,b + a\,c + b\,c)$ and suppose we want to reduce it with respect to variable a.

On the $\bullet$ branch, simplify with $a = 0$:

$$d \oplus e \oplus (0\,b + 0\,c + b\,c) \;\; = \;\; d \oplus e \oplus bc$$

On the $\circ$ branch, simplify with $a = 1$:

$$d \oplus e \oplus (1\,b + 1\,c + b\,c) \;\; = \;\; d \oplus e \oplus (b + c + b\,c) \;\; \stackrel{*}{=} \;\; d \oplus e \oplus (b + c)$$

The residual expressions are then reduced with respect to the next variable in the ordering.

The ROBDD on the left uses variable ordering $\langle d, e, a, b, c\rangle$; the one on the right uses $\langle b, a, d, e, c\rangle$. In the examples below use the '$\oplus$' identities $0 \oplus x = x \oplus 0 = x$ and $1 \oplus x = x \oplus 1 = \overline{x}$.