A “houses” proof of the Pythagorean theorem

Lindsey Kuper

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We are interested in showing that, given a right triangle, the sum of the squares of the two legs' lengths is equal to the square of the hypotenuse’s length. Here’s one such triangle:

We’ll use this triangle to illustrate the proof, but our result will apply to any right triangle.

It’s equivalent to show that the sum of the areas of the two squares whose sides are the legs is equal to the area of the square whose side is the hypotenuse. Adding those three squares to our drawing, with each sharing a side with the original triangle, results in the following figure:
Now that we have small, medium, and large squares, we can imagine two houses, a small house and a medium house. Each house consists of a square room attached to a triangular roof. We separate the original triangle into the two roofs with a short dividing line that is perpendicular to the hypotenuse:

The roof of the small house and the roof of the medium house are both right triangles, since the dividing line intersects the original triangle’s hypotenuse at a right angle. Furthermore, the small roof and the medium roof each share a non-right angle with the original triangle. So, since the small roof and the medium roof each share two angles with the original triangle, they must share their third angles with the original triangle as well, and so both are similar to the original triangle and therefore also to each other.
Finally, observe that the large square and the original triangle together form a large house:

We now have three rooms and three roofs, with our original triangle serving simultaneously as the large roof and as the small and medium roofs. We’ve already seen that the three roofs are similar triangles, and since the rooms are similar squares constructed from corresponding sides of those similar triangles, the relation that holds between the areas of the roofs must also hold between the areas of the rooms.

Since the large roof’s area is the sum of the areas of the small and medium roofs, the large room’s area is the sum of the areas of the small and medium rooms. Therefore, the areas of the two squares whose sides are the legs of the original triangle sum to the area of the square whose side is the hypotenuse, as we set out to show.