A Lattice-Based Approach to Deterministic Parallelism with Shared State

Lindsey Kuper and Ryan R. Newton
Indiana University
Bloomington, Indiana, USA

Aarhus University
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let _ = put l 3 in
  let par v = get l
    _ = put l 4
  in v
What do we want?
What do we want?

- A deterministic program is one that always produces the same observable result on multiple runs.
What do we want?

- A **deterministic** program is one that always produces the same observable result on multiple runs.

- A **deterministic-by-construction** programming model is one that only allows deterministic programs to be written.
What do we want?

- A **deterministic** program is one that always produces the same observable result on multiple runs.
- A **deterministic-by-construction** programming model is one that only allows deterministic programs to be written.
  - Examples: Kahn process networks, Intel Concurrent Collections, Haskell’s monad-par, ...
let _ = put l 3 in
let par \( v = \) get \( l \)
_ = put \( l \) 4
in \( v \)
let _ = put l 3 in
let par v = get l
  _ = put l 4
in v
let _ = put l 3 in
let par v = get l in
let _ = put l 4
in v
let _ = put l 3 in
  let par v = get l
    _ = put l 4
      in v
Disallow shared state?

let _ = put l 3 in
let par v = get l
    _ = put l 4
in v
Disallow shared state?

let _ = put l 3 in

let par v = get l

_ = put l 4

in v
Disallow shared state?

let _ = put l 3 in
let par v = get l
    _ = put l 4
    in v
Disallow multiple assignment?

\[
\begin{align*}
\text{let } &\_ = \text{put } l \ 3 \ \text{in} \\
\text{let par } &v = \text{get } l \\
\_ &\_ = \text{put } l \ 4 \\
\text{in } &v
\end{align*}
\]
Disallow multiple assignment?

let _ = put l 3 in

let par v = get l

_ = put l 4 ×

in v
A few single-assignment languages
A few single-assignment languages

- Historically:
  - Compel (Tesler and Enea, 1968)
A few single-assignment languages

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  - Compel (Tesler and Enea, 1968)
  - Id, I-Structures and IVars (Arvind et al., 1989)
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  - Id, I-Structures and IVars (Arvind et al., 1989)

- Today:
  - Intel Concurrent Collections (Budimlić et al., 2010)
    - Specifically, Featherweight CnC
A few single-assignment languages

- Historically:
  - Compel (Tesler and Enea, 1968)
  - Id, I-Structures and IVars (Arvind et al., 1989)

- Today:
  - Intel Concurrent Collections (Budimlic et al., 2010)
    - Specifically, Featherweight CnC
  - monad-par for Haskell (Marlow et al., 2011)
Disallow multiple assignment?

\[
\begin{align*}
\text{let } & \_ = \text{put } l\ 3 \text{ in} \\
\text{let par } v &= \text{get } l \\
\_ & = \text{put } l\ 4 \quad \times \\
\text{in } v
\end{align*}
\]
Deterministic programs that single-assignment forbids

```
let _ = put l 3 in
  let par v = get l
    _ = put l 3
  in v
```
Deterministic programs that single-assignment forbids

\[
\begin{align*}
\text{let } & \_ = \text{put } l \ 3 \ \text{in} \\
\text{let par } & v = \text{get } l \\
\_ & = \text{put } l \ 3 \\
\text{in } & v
\end{align*}
\]

\[
\begin{align*}
\text{let par } & \_ = \text{put } l \ (4, \bot) \\
\_ & = \text{put } l \ (\bot, 3) \\
\text{in let } & v = \text{get } l \ \text{in } v
\end{align*}
\]
Kahn process networks (Kahn, 1974)
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\[
\begin{array}{c}
\text{C}_1 \\
sto \text{C}_2 \\
sto \text{C}_3 \\
sto \text{C}_4 \\
sto \text{C}_5 \\
\end{array}
\]

\[\text{hist}(\text{in}(\text{C}_3)): [3, 0, 5, ...]\]
Kahn process networks (Kahn, 1974)

\[
\begin{align*}
\text{hist}(\text{in}(C_3)) & : [3, 0, 5, \ldots] \\
\text{hist}(\text{out}(C_3)) & : [6, 1, 120, \ldots]
\end{align*}
\]
Monotonicity
Monotonicity
Monotonicity

\[ f \text{ is monotonic iff } x \leq y \implies f(x) \leq f(y) \]
Monotonicity in KPNs

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Monotonicity in KPNs

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\[ C_1 \xrightarrow{\text{in}(C_3)} C_3 \xrightarrow{\text{out}(C_3)} C_5 \]
Monotonicity in KPNs

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Monotonicity in KPNs

\[ f \text{ is monotonic iff } x \leq y \implies f(x) \leq f(y) \]

For KPNs, the \( \leq \) relation is just \textit{prefix-of}:

\[
[3] \text{ prefix-of } [3, 0] \implies [6] \text{ prefix-of } [6, 1]
\]
\[
[3, 0] \text{ prefix-of } [3, 0, 5] \implies [6, 1] \text{ prefix-of } [6, 1, 120]
\]
\[
\vdots
\]
Monotonicity causes deterministic parallelism!
Back to single-assignment languages

let _ = put \( l_1 \) 4 in

let _ = put \( l_2 \) 3 in

let par _ = put \( l_4 \) 3

_ = put \( l_3 \) 5

in get \( l_4 \)

Store:
Back to single-assignment languages

let _ = put \( l_1 \) 4 in
let _ = put \( l_2 \) 3 in
let \( \text{par} \_ = \text{put} \ l_4 \ 3 \)
    _ = put \( l_3 \) 5
in get \( l_4 \)

Store:

| \( l_1 \) | 4 |
Back to single-assignment languages

let _ = put $l_1$ 4 in
let _ = put $l_2$ 3 in
let par _ = put $l_4$ 3
_ = put $l_3$ 5
in get $l_4$

Store:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1$</td>
<td>4</td>
</tr>
<tr>
<td>$l_2$</td>
<td>3</td>
</tr>
</tbody>
</table>
Back to single-assignment languages

let _ = put \( l_1 \) 4 in
let _ = put \( l_2 \) 3 in
let par _ = put \( l_4 \) 3
_ = put \( l_3 \) 5

in get \( l_4 \)

Store:

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</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>4</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>3</td>
</tr>
<tr>
<td>( l_3 )</td>
<td>5</td>
</tr>
</tbody>
</table>
Back to single-assignment languages

let _ = put \( l_1 \) 4 in

let _ = put \( l_2 \) 3 in

let par _ = put \( l_4 \) 3

_ = put \( l_3 \) 5

in get \( l_4 \)

Store:

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<td>( l_1 )</td>
<td>4</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>3</td>
</tr>
<tr>
<td>( l_3 )</td>
<td>5</td>
</tr>
<tr>
<td>( l_4 )</td>
<td>3</td>
</tr>
</tbody>
</table>
Back to single-assignment languages

let _ = put \( l_1 \rightarrow 4 \) in

let _ = put \( l_2 \rightarrow 3 \) in

let par _ = put \( l_4 \rightarrow 3 \)

_ = put \( l_3 \rightarrow 5 \)

in get \( l_4 \)

Store:

| \( l_1 \) | 4 |
| \( l_2 \) | 3 |
| \( l_3 \) | 5 |
| \( l_4 \) | 3 |

For stores, the \( \leq \) relation is \( \subseteq \):

\[
\{ l_1 \rightarrow 4, \ l_2 \rightarrow 3 \} \subseteq \{ l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_3 \rightarrow 5 \} \implies
\{ l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_4 \rightarrow 3 \} \subseteq \{ l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_3 \rightarrow 5, \ l_4 \rightarrow 3 \}
\]
Generalizing our notion of monotonicity

For stores, the $\leq$ relation is $\subseteq$:

$$\{l_1 \rightarrow 4, l_2 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4, l_2 \rightarrow 3, l_3 \rightarrow 5\} \implies$$

$$\{l_1 \rightarrow 4, l_2 \rightarrow 3, l_4 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4, l_2 \rightarrow 3, l_3 \rightarrow 5, l_4 \rightarrow 3\}$$

- Given stores $S$ and $S'$, we say that $S \leq S'$ iff:
  - $\text{dom}(S) \subseteq \text{dom}(S')$, and
  - for all locations $l$ in $\text{dom}(S)$, $S(l) = S'(l)$
Generalizing our notion of monotonicity

For stores, the \( \leq \) relation is \( \subseteq \):

\[
\{l_1 \rightarrow 4, \ l_2 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_3 \rightarrow 5\} \implies \\
\{l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_4 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4, \ l_2 \rightarrow 3, \ l_3 \rightarrow 5, \ l_4 \rightarrow 3\}
\]

- Given stores \( S \) and \( S' \), we say that \( S \leq S' \) iff:
  - \( \text{dom}(S) \subseteq \text{dom}(S') \), and
  - for all locations \( l \) in \( \text{dom}(S) \), \( S(l) \leq S'(l) \) (user-specified)
Idea: restrict reads to “threshold” reads

let _ = put l 3 in

let par \( v = \) get l 4

\_ = \text{put } l 4

in \( v \)
Idea: restrict reads to “threshold” reads

```
let _ = put l 3 in
  let par v = get l 4
  _ = put l 4
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Idea: restrict reads to “threshold” reads

let _ = put l 3 in
  let par v = get l 4
    _ = put l 4
  in v

let _ = put l 3 in
  let par v = get l 4
    _ = put l 4
    _ = put l 5
  in v
Idea: restrict reads to “threshold” reads

\[
\begin{align*}
\text{let } _ &= \text{put } l \ 3 \ \text{in} \\
\text{let par } v &= \text{get } l \ 4 \\
\_ &= \text{put } l \ 4 \\
\text{in } v \\
\text{let } _ &= \text{put } l \ 3 \ \text{in} \\
\text{let par } v &= \text{get } l \ 4 \\
\_ &= \text{put } l \ 4 \\
\_ &= \text{put } l \ 5 \\
\text{in } v
\end{align*}
\]
Idea: restrict reads to “threshold” reads

\[
\begin{align*}
&\text{let } _ = \text{put } l \ 3 \ \text{in} \\
&\quad \text{let par } v = \text{get } l \ 4 \\
&\quad \quad _ = \text{put } l \ 4 \\
&\quad \text{in } v \\
&\text{let } _ = \text{put } l \ 3 \ \text{in} \\
&\quad \text{let par } v = \text{get } l \ 4 \\
&\quad \quad _ = \text{put } l \ 4 \\
&\quad \quad _ = \text{put } l \ 5 \\
&\quad \text{in } v
\end{align*}
\]
Idea: restrict reads to “threshold” reads

let _ = put l 3 in
  let par v = get l 4
    _ = put l 4
  in v

let _ = put l 3 in
  let par v = get l 4
    _ = put l 4
    _ = put l 5
  in v

return 4
Monotonically increasing writes
+ threshold reads
= deterministic parallelism
Parameterizing our language: “LVars”

IVar

Pair of IVars

Counter
Parameterizing our language: “LVars”

Pair of IVars
Parameterizing our language: “LVars”

Pair of IVars
Parameterizing our language: “LVars”

Pair of LVars

getSnd "tripwire" getFst
Parameterizing our language: “LVars”

let $p = \text{new}$ in
  let $\_ = \text{put} \ p \{(⊥, 4)\}$ in
    let $\text{par} \ v_1 = \text{getFst} \ p$
    $\_ = \text{put} \ p \{(3, 4)\}$
  in ... $v_1$ ...

Pair of IVars

getch "tripwire"
getch
Two take-aways

Monotonicity causes deterministic parallelism

Monotonically increasing writes
+ threshold reads
= deterministic parallelism
More in our paper draft
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- Complete syntax and semantics
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- Proof of determinism
  - A “frame-rule-like” property
  - Location renaming is surprisingly tricky!
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- Subsuming existing models
  - KPNs, CnC, monad-par
More in our paper draft

- Complete syntax and semantics
- Proof of determinism
  - A “frame-rule-like” property
  - Location renaming is surprisingly tricky!
- Subsuming existing models
  - KPNs, CnC, monad-par
- Support for controlled nondeterminism
  - “probation” state
Tak!

Email: lkuper@cs.indiana.edu
Twitter: @lindsey
Web: cs.indiana.edu/~lkuper
Research group: lambda.cs.indiana.edu