Homework Assignment #1

Assigned: Tuesday 09/01/2015; Due: Tuesday 09/15/2015 by 11:59pm through Oncourse.

(total: 120 points)

Problem 1 (10 points) Consider the set of twelve functions shown below. Rank them according to their order of growth. Use $O$, $\omega$, or $\Theta$ operators.

\[
\begin{align*}
&f_1(n) = n(n + 2) & &f_7(n) = 2^n \\
&f_2(n) = 2015n^2 & &f_8(n) = 0.0001n^3 \\
&f_3(n) = \log_2 n & &f_9(n) = 100n^2 \\
&f_4(n) = 2^{n-1} & &f_{10}(n) = n! \\
&f_5(n) = (n - 1)! & &f_{11}(n) = \ln(\ln n) \\
&f_6(n) = \log_2 n^2 & &f_{12}(n) = (\log_2 n)^2
\end{align*}
\]

Problem 2 (10 points) Consider the following algorithm:

Algorithm $f(n)$
\[
\begin{align*}
&\text{if } n = 1 \\
&\quad \text{return } 1 \\
&\text{else} \\
&\quad \text{return } f(n - 1) + 3 \cdot n - 1 \\
&\end{align*}
\]

a) (5 points) Determine what the function computes
b) (5 points) Write and solve the recurrence for this program that computes the number of basic operations as a function of $n$.

Problem 3 (5 points) Prove or disprove the following statement

\[n! = o(n^n)\]

Problem 4 (12 points) Use the master theorem to give asymptotic bounds for the recurrences below

\[
\begin{align*}
a) & & (4 \text{ points}) & & T(n) = 8T(n/2) + n \\
b) & & (4 \text{ points}) & & T(n) = 8T(n/2) + n^3 \\
c) & & (4 \text{ points}) & & T(n) = 8T(n/2) + n^6 \\
d) & & (4 \text{ points}) & & T(n) = 4T(n/2) + \sqrt{n}
\end{align*}
\]
Assume $T(1) = 1$ in all cases.

**Problem 5** (5 points) Use the variable change method (see textbook) to solve the following recurrence

$$T(n) = \begin{cases} 3T\left(\sqrt{n}\right) + \log n & n > 1 \\ 1 & n = 1 \end{cases}$$

**Problem 6** (15 points) Given is an array of $n$ integers. Propose an algorithm that finds a consecutive set of $m$ elements with the largest sum, where $m \leq n$. Analyze the run time of your algorithm and compute $T(n)$.

**Problem 7** (20 points) Show the correctness of the following expression

a) (2 points) $10n^2 + 30n = O(n^3)$
b) (3 points) $10n^2 + 30n \neq \Omega(n^3)$
c) (5 points) $2^n = \Omega(5\ln n)$
d) (5 points) $(\ln n)^3 = o(n^{0.25})$  
e) (5 points) $f^2(n) = \Omega(f(n))$

**Problem 8** (20 points) Give asymptotic run times for the recurrences below

a) (5 points) $T(n) = T(\sqrt{n}) + n$
b) (5 points) $T(n) = T(n - 1) + n^2$
c) (5 points) $T(n) = T(n/4) + T(3n/4) + n^2$
d) (5 points) $T(n) = 4T(n/4) + 1$

Assume $T(1) = 1$ in all cases.

**Problem 9** (23 points) Implement functions `insertion_sort()` and `merge_sort()` in a programming language of your choice (pick C/C++, Java, Python, or MATLAB). Each function should return the number of comparisons performed in it. First, make sure your input array is properly sorted. Then implement a wrapper function that calls each function for an input array of $n$ randomly generated integers. Vary input sizes as $n = 5, 10, 15, ..., 100$ and call each function at least 100 times for each $n$ to get stable estimates of the number of comparisons. Draw a graph of average number of comparisons (x-axis should be $n$ and the y-axis should be the average number of comparisons over 100 random starts). Draw relevant conclusions about the agreement between theoretical complexity and observed results.
Homework policies:

This assignment is strictly individual. All code (if applicable) should be turned in when you submit your assignment (as hard copy).

Policy for late submission assignments: Unless there are legitimate circumstances, late assignments will be accepted up to 5 days after the due date and graded using the following rule:

- on time: your score × 1
- 1 day late: your score × 0.9
- 2 days late: your score × 0.7
- 3 days late: your score × 0.5
- 4 days late: your score × 0.3
- 5 days late: your score × 0.1

For example, this means that if you submit 3 days late and get 80 points for your answers, your total number of points will be $80 \times 0.5 = 40$ points.

All the sources used for problem solution must be acknowledged, e.g. web sites, books, research papers, personal communication with people, etc. Academic honesty is taken seriously; for detailed information see Indiana University Code of Student Rights, Responsibilities, and Conduct.

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Good luck!